6.3: Logarithmic Functions

To solve for x in exponential equations such as $10^x = 85$, change exponential equation into a logarithmic equation.

Exponential Form		Logarithmic Form
$10^3 = 1000$		$3 = \log_{10} 1000$
	exponent	
	base	

*Base stays a base, flip-flop other two numbers.

Equivalent Exponential and Logarithmic Forms : For any positive base b, where $b \neq 1$,

$$b^x = y \text{ iff } x = \log_b y$$

Logs are exponents

Exponential Form	$2^5 = 32$	$10^3 = 1000$	$3^{-2} = \frac{1}{9}$	$16^{\frac{1}{2}} = 4$
Logarithmic Form	$5 = \log_2 32$	$\log_{10} 1000 = 3$	$-2 = \log_3 \frac{1}{9}$	$\log_{16} 4 = \frac{1}{2}$

To solve $10^x = 85$ for x ...

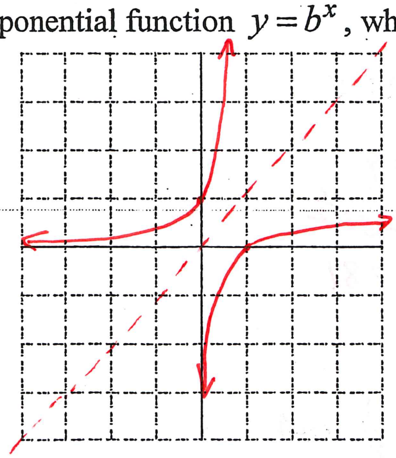
1) Write in logarithmic form: $x = \log_{10} 85$
2) We can evaluate logarithms with base 10 using the **LOG** key on calculator.
   $x = \log_{10} 85$ becomes $x = 1.929$

Solve $10^x = 14.5$ for x.

$\log_{10} 14.5 = x$

$1.161 = x$

The **Logarithmic Function** $y = \log_b x$ with base b, or $x = b^y$, is the **inverse** of the exponential function $y = b^x$, where $b \neq 1$ and $b > 0$.



Graph $y = x$ (line of reflection), $y = 10^x$, and $y = \log_{10} x$ and sketch on the provided graph.

$y = 10^x$	$y = \log_{10} x$
Domain: all reals	**Domain:** $x > 0$
Range: $y > 0$	**Range:** all reals

One-to-One Property of Exponents: If $b^x = b^y$, then $x = y$.

*If bases are equal, then exponents are equal.
*If exponents are equal, then bases are equal.

Solving logarithmic functions **without** base 10

- Write equivalent exponential form
- Change a base or exponent (goal: to create equal bases or equal exponents)
  If the variable is the exponent then create equal bases
  If the variable is the base then create equal exponents
- Apply one-to-one property

**Can check answer by graphing the exponential form on calculator.
→Enter one side in y_1 and the other in y_2
→Find intersection point

Examples: Solve equations for given variable.

1) $V = \log_{125} 5$

$125^V = 5$ or $125^V = 125^{\frac{1}{3}}$

$(5^3)^V = 5^1$

$5^{3V} = 5^1$

$3V = 1$

$V = \frac{1}{3}$

2) $5 = \log_v 32$

$V^5 = 32$

$V^5 = 2^5$

$V = 2$

3) $4 = \log_3 v$

$3^4 = V$

$81 = V$

4) $v = \log_4 64$

$4^V = 64$

$4^V = 4^3$

$V = 3$

5) $2 = \log_v 25$

$V^2 = 25$

$V^2 = 5^2$

$V = 5$

6) $6 = \log_3 v$

$3^6 = V$

$729 = V$

7) $r = \log_2 1$

$2^r = 1$

$2^r = 2^0$

$r = 0$

8) $\frac{1}{2} = \log_y 9$

$y^{\frac{1}{2}} = 9$

$y^{\frac{1}{2}} = 81^{\frac{1}{2}}$

$y = 81$

9) $3 = \log_7 d$

$7^3 = d$

$343 = d$

10) $\log_x \frac{1}{81} = -4$

$x^{-4} = \frac{1}{81}$

$x^{-4} = 3^{-4}$

$x = 3$